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LETTER TO THE EDITOR

# Conformal properties of primary fields in a $q$-deformed theory

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**Abstract.** We examine some of the standard features of primary fields in the framework of a $q$-deformed conformal field theory. By introducing a $q$-OPE between the energy-momentum tensor and a primary field, we derive the $q$-analogue of the conformal Ward identities for correlation functions of primary fields. We also obtain solutions to these identities for the two-point function.

In recent years, there has been growing interest in the study of quantized universal enveloping algebras. Loosely called quantum groups, they first appeared in the study of the quantum Yang–Baxter equations related to the inverse scattering problem [1]. Subsequently, it was shown that they can be obtained from representations of mathematical structures called quasi-triangular Hopf algebras [2]. These structures, which often depend on a parameter $q$, can be regarded as $q$-deformations of Lie algebras in the sense that as $q \to 1$ the algebra reduces to the usual Lie algebra.

Explicit realizations of some of these quantum groups have been obtained by many authors [2–4]. For instance the Jordan–Schwinger approach often used in the study of angular momentum algebra has been suitably generalized to give bosonic ($q$-oscillator) representations of the quantum group $\mathrm{SU}_q(2)$ [4]. More recently, Curtright and Zachos [5] have constructed a $q$-analogue of the centreless Virasoro algebra by using a differential realization of $\mathrm{SU}_q(1,1)$ (see also [6]). The central extension to this algebra has been furnished by Aizawa and Sato [7]. In fact, they have also found a $q$-deformed operator product expansion (OPE) between two energy–momentum tensors which realizes this algebra. This naturally paves the way for a $q$-deformed conformal field theory.

In this letter we study the properties of primary fields in the spirit of [7]. Here we re-examine some of the well known issues pertaining to standard conformal field theory (CFT) [8] in the context of such a $q$-deformed theory. In particular, we introduce a $q$-OPE of the energy–momentum tensor with a primary field which extends the $q$-OPE of [7] to primary fields of arbitrary conformal weights. The deformation reflected in this $q$-OPE is shown to be equivalent to the one used by Chaichan *et al* [9]. Using arguments paralleling those used in standard CFT, we obtain the $q$-analogue of the conformal Ward identity and the projective Ward identities for correlation functions of primary fields. In particular, for the two-point function, it is shown that these Ward identities do not uniquely determine it when $|q| = 1$. Using the $q$-OPE we also realize

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the algebra between the modes of the energy–momentum tensor and those of a holomorphic primary field.

We begin by summarizing some basic features of standard CFTs [8, 10] that will be used or modified later. Consider a primary field $\Phi(z, \bar{z})$ with conformal weights $h$, $\bar{h}$. It is defined by its transformation under $z \to z' = f(z)$, $\bar{z} \to \bar{z}' = \bar{f}(\bar{z})$:

$$\Phi(z, \bar{z}) \to \Phi'(z, \bar{z}) = (\partial f)^h (\overline{\partial f})^{\bar{h}} \Phi(f(z), \bar{f}(\bar{z})) \tag{1}$$

where $f(z)$ and $\bar{f}(\bar{z})$ are arbitrary holomorphic and antiholomorphic functions respectively. When the transformation is infinitesimal, i.e. $f(z) = z + \varepsilon(z)$ and $\bar{f}(\bar{z}) = \bar{z} + \bar{z} + \bar{\varepsilon}(\bar{z})$, then

$$\Phi'(z, \bar{z}) = \Phi(z, \bar{z}) + \Delta_{\varepsilon, \bar{\varepsilon}} \Phi(z, \bar{z}) \tag{2a}$$

with

$$\Delta_{\varepsilon, \bar{\varepsilon}} \Phi(z, \bar{z}) = (h \partial \varepsilon + \varepsilon \partial) \Phi(z, \bar{z}) + (\overline{h \partial \bar{\varepsilon}} + \bar{\varepsilon} \bar{\partial}) \Phi(z, \bar{z}). \tag{2b}$$

In particular when $\varepsilon(z) = \varepsilon_n z^{n+1}$ and $\bar{\varepsilon}(\bar{z}) = \bar{\varepsilon}_n \bar{z}^{n+1}$ where $\varepsilon_n$ and $\bar{\varepsilon}_n$ are small constants, we have

$$\Delta_n \Phi(z, \bar{z}) = \varepsilon_n \delta_n \Phi(z, \bar{z}) + \bar{\varepsilon}_m \bar{\delta}_m \Phi(z, \bar{z}) \tag{3a}$$

where

$$\delta_n \Phi(z, \bar{z}) = (z\partial + h(n+1) - n) z^n \Phi(z, \bar{z}) \tag{3b}$$

$$\bar{\delta}_n \Phi(z, \bar{z}) = (\bar{z}\bar{\partial} + \bar{h}(n+1) - n) \bar{z}^n \Phi(z, \bar{z}). \tag{3c}$$

In the following, we will only consider the holomorphic terms with similar results holding for the antiholomorphic ones.

*In a quantum theory, the variation in* $\Phi(z, \bar{z})$ *is implemented by the* 'equal-time' commutator:

$$\delta_n \Phi(w, \bar{w}) = \left[ \oint_{C_0} \frac{\mathrm{d}z}{2\pi \mathrm{i}} z^{n+1} T(z), \Phi(w, \bar{w}) \right] \tag{4}$$

where $T(z)$ is the holomorphic component of the energy–momentum tensor. On the $z$-plane, different times correspond to concentric circles of different radii and the notion of time ordering is replaced by that of radial ordering:

$$R(A(z)B(w)) = \begin{cases} A(z)B(w) & |z| > |w| \\ B(w)A(z) & |z| < |w|. \end{cases} \tag{5}$$

In this scheme the 'equal-time' commutator is given by [10]

$$\begin{aligned} &\oint_{C_0} \frac{\mathrm{d}z}{2\pi \mathrm{i}} z^{n+1} [T(z), \Phi(w, \bar{w})] \\ &\qquad = \left( \oint_{|z|>|w|} - \oint_{|z|<|w|} \right) \frac{\mathrm{d}z}{2\pi \mathrm{i}} z^{n+1} R(T(z)\Phi(w, \bar{w})) \\ &\qquad = \oint_{C_P} \frac{\mathrm{d}z}{2\pi \mathrm{i}} z^{n+1} R(T(z)\Phi(w, \bar{w})) \end{aligned} \tag{6}$$

where the last integral is taken around all the poles in the OPE of $T(z)\Phi(w, \bar{w})$ which we assume are located on the $|z| = |w|$ contour. Indeed, by comparison with (3$b$) one can infer that

$$T(z)\Phi(w, \bar{w}) = \frac{h\Phi(w, \bar{w})}{(z-w)^2} + \frac{\partial \Phi(w, \bar{w})}{(z-w)} + \text{regular terms}. \tag{7}$$

Now a $q$-deformation of the theory is achieved by replacing (3$b$) and (3$c$) by the corresponding $q$-analogues. For this purpose we consider the deformation as defined by Chaichan *et al* [9]:

$$\delta_n\Phi(z,\bar{z})\to\delta_n^q\Phi(z,\bar{z})=[z\partial+h(n+1)-n]z^n\Phi(z,\bar{z}) \tag{8}$$

which essentially replaces the bracket in (3$b$) by a $q$-bracket defined by

$$[x]=\frac{q^x-q^{-x}}{q-q^{-1}}. \tag{9}$$

Equation (8) thus serves as a definition of a primary field in a $q$-deformed theory. We will now implement this variation as an 'equal-time' commutator as in (6). To this end, we introduce a $q$-OPE of $T(z)$ with $\Phi(w,\bar{w})$, which we write as

$$\begin{aligned}(T(z)\Phi(w,\bar{w}))_q &=\frac{[h/2]}{(z-w)}\left\{\frac{\Phi(wq^{-1},\bar{w})}{zq^{h/2}-wq^{-h/2}}+\frac{\Phi(wq,\bar{w})}{zq^{-h/2}-wq^{h/2}}\right\}\\ &\quad+\frac{1}{(z-w)}\partial_w^q\Phi(w,\bar{w})+\text{regular terms}\end{aligned} \tag{10}$$

where $\partial_w^q$ is the $q$-analogue of the derivative:

$$\partial_w^q f(w)=\frac{f(wq)-f(wq^{-1})}{w(q-q^{-1})}. \tag{11}$$

Using this definition for the $q$-derivative, we can also rewrite the $q$-OPE as

$$(T(z)\Phi(w,\bar{w}))_q=\frac{1}{w(q-q^{-1})}\left\{\frac{\Phi(wq,\bar{w})}{z-wq^h}-\frac{\Phi(wq^{-1},\bar{w})}{z-wq^{-h}}\right\}+\text{regular terms} \tag{12}$$

which shows that it is singular at the points $z=wq^{\pm h}$. It is easy to verify that the above OPE leads to the correct variation in $\Phi$, by evaluating the integral in (6) with $C_P$ taken as a contour encircling the points $wq^h$ and $wq^{-h}$. Before proceeding further, let us make a few observations.

(1) It is evident from the above expression that there are poles present at two points rather than one. These poles are both of order 1, unlike the undeformed case where the $z=w$ pole is of order 2. It is interesting to note, however, that in the limit $q\to 1$ these poles will coalesce to form a pole of order 2 at $z=w$. In fact, in the limit $q\to 1$, our $q$-OPE reduces to the standard one (7).

(2) Recall that the 'equal-time' commutator in (6) was evaluated as a difference of two integrals with contours which are concentric and close to the $|z|=|w|$ contour but one having radius $|z|>|w|$ and the other $|z|<|w|$. They combine into a single contour which is taken to be a small circle centred around the singular point ($z=w$). For this scheme to be applicable here, we must require that the two poles lie on the $|z|=|w|$ contour, since otherwise this poles will not make any contributions to the integral. This means that we must restrict ourselves to the case when $|q|=1$, i.e. $q$ should be taken as a pure phase ($q-e^{i\alpha\pi}$).

(3) It is worth noting that the variation in $\Phi$ obtained by our $q$-OPE is similar to the one used by Chaichan *et al* only for the case $\varepsilon(z)=z^{n+1}$. For arbitrary $\varepsilon(z)$, their variation is assumed to be of the form

$$\delta_\varepsilon^q\Phi(z,\bar{z})=\varepsilon(z)^{1-h}\partial_z^q(\varepsilon(z)^h\Phi(z,\bar{z})) \tag{13}$$

while ours is given by

$$\delta_\varepsilon^q \Phi(z,\bar{z}) = \oint_{C_p} \frac{d\xi}{2\pi i} \varepsilon(\xi) R(T(\xi)\Phi(z,\bar{z}))_q$$
$$= \varepsilon(zq^h)\partial_z^q \Phi(z,\bar{z}) + [h]\partial_z^{q^h} \varepsilon(z)\Phi(zq^{-1},\bar{z}). \tag{14}$$

Both, however, reduce to (8) when $\varepsilon(z)$ is taken to be $z^{n+1}$.

(4) When $h=2$, our expression is similar to the one given by Aizawa and Sato [7] for the OPE of two energy momentum tensors when the central charge in their expression is taken as zero.

With the $q$-OPE defined as above, we can write down the $q$-Ward identities for the correlation functions of primary fields. We begin by considering the action of the generator of infinitesimal conformal transformations on the correlation of $n$ primary fields $\{\Phi_i(w_i,\bar{w}_i)\}$ with corresponding conformal weights $h_i$, $\bar{h}_i$ $(i=1,2,\ldots n)$:

$$\left\langle \oint_{C_0} \frac{dz}{2\pi i} \varepsilon(z) T(z) \Phi_1(w_1,\bar{w}_1)\ldots\Phi_n(w_n,\bar{w}_n)\right\rangle_q. \tag{15}$$

(The contour $C_0$ encircles all the points $\{w_i q^{hk} \mid k=\pm 1\}_{i=1,2,\ldots,n}$.) In the above expression, the correlation function $\langle\ldots\rangle_q$ is taken relative to the 'in' ($|0\rangle_q$) and the 'out' (${}_q\langle 0|$) vacuums which are defined by requiring that

$$L_m|0\rangle_q = 0 \qquad m \geqslant -1 \tag{16a}$$

$${}_q\langle 0|L_m = 0 \qquad m \leqslant 1 \tag{16b}$$

where

$$L_m = \oint_{C_0} \frac{dz}{2\pi i} z^{m+1} T(z) \qquad m \in \mathbb{Z} \tag{17}$$

are modes in the expansion,

$$T(z) = \sum_{m\in\mathbb{Z}} L_m z^{-m-2}. \tag{18}$$

Note that conditions (16$a$) and (16$b$) ensure the regularity of $T(z)|0\rangle_q$ and its adjoint at $z=0$ and $z=\infty$. By analyticity, the contour $C_0$ in expression (15) can be deformed to a sum of $n$ contours with each contour $C_i$ surrounding the points, $\{w_i q^h, w_i q^{-h}\}$. Then as a consequence of the $q$-OPE, we have

$$\oint_{C_0} \frac{dz}{2\pi i} \varepsilon(z) \langle T(z)\Phi_1(w_1,\bar{w}_1)\ldots\Phi_n(w_n,\bar{w}_n)\rangle_q$$
$$= \sum_{i=1}^n \left\langle \Phi_1(w_1,\bar{w}_1)\ldots \oint_{C_i}\frac{dz}{2\pi i}\varepsilon(z)(T(z)\Phi_i(w_i,\bar{w}_i))_q \ldots \Phi_n(w_n,\bar{w}_n)\right\rangle_q$$
$$= \sum_{i=1}^n \oint_{C_0} \frac{dz}{2\pi i} \varepsilon(z) \mathscr{L}^{h_i}_{z;w_i} \langle \Phi_1(w_1,\bar{w}_1)\ldots\Phi_n(w_n,\bar{w}_n)\rangle_q \tag{19}$$

where the differential operator $\mathscr{L}^{h_i}_{z;w_i}$ is given by

$$\mathscr{L}^{h_i}_{z;\omega_i} \equiv \frac{1}{(z-w_i)}\left\{[h_i/2]\left(\frac{q^{-w_i\partial_{w_i}}}{zq^{h_i/2}-w_i q^{-h_i/2}} + \frac{q^{w_i\partial_{w_i}}}{zq^{-h_i/2}-w_i q^{h_i/2}}\right) + \partial^q_{w_i}\right\}. \tag{20}$$

Furthermore, since $\varepsilon(z)$ is arbitrary, we can write

$$\langle T(z)\Phi_1(w_1,\bar{w}_1)\ldots\Phi_n(w_n,\bar{w}_n)\rangle_q = \sum_{i=1}^n \mathscr{L}^{h_i}_{z;w_i}\langle\Phi_1(w_1,\bar{w}_1)\ldots\Phi_n(w_n,\bar{w}_n)\rangle_q \tag{21}$$

which is the unintegrated form of the $q$-Ward identity. It is easy to see that it reduces to the usual one as $q \to 1$.

Next let us consider the $q$-analogue of the projective Ward identities. From (16$a$) and (16$b$) it is easy to see that the generators $L_{0,\pm 1}$ annihilate both the 'in' and the 'out' vacuums. On substituting $\varepsilon(z) = z^{m+1}$ for $m = -1, 0, 1$ into (19) and integrating, we have

$$\sum_{i=1}^{n} w_i^{-1}[w_i \partial_{w_i}]\langle \Phi_1(w_1, \bar{w}_1) \ldots \Phi_n(w_n, \bar{w}_n)\rangle_q = 0 \tag{22a}$$

$$\sum_{i=1}^{n} [w_i \partial_{w_i} + h_i]\langle \Phi_1(w_1, \bar{w}_1) \ldots \Phi_n(w_n, \bar{w}_n)\rangle_q = 0 \tag{22b}$$

$$\sum_{i=1}^{n} w_i[w_i \partial_{w_i} + 2h_i]\langle \Phi_1(w_1, \bar{w}_1) \ldots \Phi_n(w_n, \bar{w}_n)\rangle_q = 0 \tag{22c}$$

for any $n$-point function. These are the $q$-analogues of the projective Ward identities. Now, it is well known that in standard CFT the two-point and three-point functions are severely constrained by the Ward identities. In fact, they are uniquely determined up to a normalization constant. The situation for the $q$-deformed case is not quite the same. Here when $|q| = 1$ the $q$-Ward identities do not uniquely specify them as we will illustrate below. For this purpose assume an ansatz for the correlation function of two primary fields $\Phi_1(w_1, \bar{w}_1)$, $\Phi_2(w_2, \bar{w}_2)$ with conformal weight $h_1$, $h_2$ respectively to be of the form

$$\langle \Phi_1(w_1, \bar{w}_1)\Phi_2(w_2, \bar{w}_2)\rangle_q = \frac{1}{(w_1 - w_2)_q^n (\bar{w}_1 - \bar{w}_2)_q^{\bar{n}}} \qquad |w_1| > |w_2| \tag{23}$$

where

$$(w_1 - w_2)_q^n = \prod_{k=1}^{n} (w_1 - w_2 q^{n-2k+1}) = \sum_{k=1}^{n} \frac{[n]!}{[n-k]![k]!} w_1^{n-k}(-w_2)^k \tag{24}$$

is the $q$-analogue of the distance function $(w_1 - w_2)^n$ [7]. On substitution into (22$a$), (22$b$) and (22$c$) we obtain the following conditions:

$$[h_1 - n] + [h_2] = 0 \tag{25a}$$

$$[h_2 - n] + [h_1] = 0 \tag{25b}$$

$$[2h_1 - n] = 0 \tag{25c}$$

$$[2h_2 - n] = 0 \tag{25d}$$

$$[2h_2] - [2h_1] = 0. \tag{25e}$$

Apart from the obvious solution

$$h_1 = h_2 = n/2 \tag{26}$$

we also have for $q = e^{i\pi\alpha}$,

$$h_1 = n/2 + k/\alpha \tag{27a}$$

$$h_2 = n/2 + l/\alpha \tag{27b}$$

where $k$ and $l$ are arbitrary integers which are either both even or both odd. Adding the two we have

$$n = h_1 + h_2 - (k + l)/\alpha \tag{28}$$

and this means that $n$ which characterizes the solution is not unique by virtue of the fact that $k$ and $l$ are arbitrary.

It is also interesting to study the commutator algebra (or rather 'quommutator') of the generators $\{L_n\}$ with the modes $\{\phi_m\}$ of a primary field. Consider a holomorphic primary field with conformal weights $(h, 0)$,

$$\Phi(w) = \sum_{m \in \mathbb{Z}-h} \phi_m w^{-m-h} \tag{29}$$

with the modes $\{\phi_m\}$ satisfying

$$\phi_m = \oint_{C_0} \frac{\mathrm{d}w}{2\pi \mathrm{i}} w^{m+h-1} \Phi(w). \tag{30}$$

Here we would like to evaluate the bracket

$$[L_n, \phi_m] \equiv (L_n \phi_m)_q - (\phi_m L_n)_q \tag{31}$$

where the terms $(\,)_q$ are defined via the $q$-product of two field operators $A(z)$ and $B(w)$ [7]:

$$(A(z)B(w))_q \equiv A(zq)B(wq^{-1}). \tag{32}$$

For instance, we have (following [7])

$$\begin{aligned}(L_n\phi_m)_q &= \oint_{C_1} \frac{\mathrm{d}z}{2\pi \mathrm{i}} \oint_{C_2} \frac{\mathrm{d}w}{2\pi \mathrm{i}} \oint_{C_2} \frac{\mathrm{d}w}{2\pi \mathrm{i}} z^{n+1} w^{m+h-1} (T(z)\Phi(w))_q \\ &= \oint_{C_1} \frac{\mathrm{d}z}{2\pi \mathrm{i}} \oint_{C_2} \frac{\mathrm{d}w}{2\pi \mathrm{i}} z^{n+1} w^{m+h-1} T(zq)\Phi(wq^{-1}) \\ &= \oint_{C_1} \frac{\mathrm{d}z}{2\pi \mathrm{i}} \oint_{C_2} \frac{\mathrm{d}w}{2\pi \mathrm{i}} z^{n+1} w^{m+h-1} \sum_k L_k (zq)^{-k-2} \sum_l \phi_l (wq^{-1})^{-l-h} \\ &= q^{m-n+h-2} L_n \phi_m \end{aligned} \tag{33}$$

where $C_1$ and $C_2$ are contours about the origin such that $C_2 \subset C_1$. Similarly

$$(\phi_m L_n)_q = q^{-(m-n+h-2)} \phi_m L_n. \tag{34}$$

Then by combining (33) and (34), the bracket in (31) can be re-expressed as

$$[L_n, \phi_m] \equiv q^{m-n+h-2} L_n \phi_m - q^{-(m-n+h-2)} \phi_m L_n. \tag{35}$$

To evaluate this bracket, we use the $q$-OPE:

$$\begin{aligned}[L_n, \phi_m] &= \oint_{C_0} \frac{\mathrm{d}z}{2\pi \mathrm{i}} \oint_{C_P} \frac{\mathrm{d}w}{2\pi \mathrm{i}} z^{n+1} w^{m+h-1} R(T(z)\Phi(w))_q \\ &= [n(h-1) - m] \phi_{n+m} \end{aligned} \tag{36}$$

which gives the 'quommutator' of $L_n$ with $\phi_m$,

$$q^{m-h+h-2} L_n \phi_m - q^{-(m-n+h-2)} \phi_m L_n = [n(h-1) - m] \phi_{n+m}. \tag{37}$$

Again, we can see that this reduces to the standard result as $q \to 1$. It is also interesting to note that if we identify $\phi_m$ with $L_m$ with $h = 2$ then the above algebra corresponds to the $q$-deformed centreless Virasoro algebra,

$$q^{m-n} L_n L_m - q^{-(m-n)} L_m L_n = [n-m] L_{n+m} \tag{38}$$

proposed by Curtright and Zachos [5].

Finally a few comments on the primary and descendant states. We define the primary state corresponding to a primary field $\Phi(w, \bar{w})$ of weights $(h, \bar{h})$ as

$$|h, \bar{h}\rangle_q = \lim_{w, \bar{w} \to 0} \Phi(w, \bar{w})|0\rangle_q \tag{39}$$

in close analogy with the standard case. In particular, for a holomorphic field with weights $(h, 0)$ the primary state $|h\rangle_q \equiv |h, 0\rangle_q$ can also be defined as

$$|h\rangle_q = \phi_{-h}|0\rangle_q. \tag{40}$$

Note that the modes $\{\phi_m\}$ for $m \geqslant -h+1$ must annihilate the 'in' vacuum as a requirement for the regularity of $\Phi(w)|0\rangle_q$ at $w = 0$. Using this fact together with (16*a*) and (37) we have

$$L_n|h\rangle_q = 0 \qquad \text{for } n > 0 \tag{41}$$

and

$$L_0|h\rangle_q = q^2[h]|h\rangle_q. \tag{42}$$

The $q$-descendant states are then constructed by subjecting the primary states to operations of $L_n$'s for $n < 0$:

$$|h; k_1, k_2 \ldots k_m\rangle_q = L_{-k_1}L_{-k_2} \ldots L_{-k_m}|h\rangle_q. \tag{43}$$

In passing, we would like to remark that it would also be interesting to study the conformal properties of secondary fields which give rise to the above $q$-descendant states. These together with the primary fields would then constitute a basis for the study of $q$-string theory.

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